

Worksheet 5

Courant Institute of Mathematical Sciences

1. Find y'' by implicit differentiation:

$$4x^3 + 3y^3 = 5$$

2. Find equations of both the tangent lines to the ellipse $x^2 + 9y^2 = 81$ that pass through the point $(27, 3)$.
3. Find the derivative of $y = \cot^2(\sin(\theta))$.
4. Find equations of both tangent lines to the ellipse $x^2 + 4y^2 = 36$ that pass through the point $(12, 3)$.
5. (a) Write the equation for $L(x)$, the linear approximation to a differentiable function $f(x)$, given $f(a)$ where a is close in proximity to x . When is $L(x)$ exactly equal to $f(x)$?
- (b) Use the previous equation to find an approximation for the difference between $f(x)$ and $f(a)$ (that is, $f(x) - f(a)$).
- (c) Connect this approximation to the differential, and explain how one can use this to estimate changes in $f(x)$ in terms of changes in x .
6. Find the derivatives of the following:
- (a) $F(x) = (4x - x^2)^{100}$
- (b) $f(z) = \frac{1}{z^2+1}$
- (c) $F(v) = \left(\frac{v}{v^3+1}\right)^6$
- (d) $y = [x + (x + \sin^2(x))^3]^4$
7. If g is a twice differentiable function and $f(x) = xg(x^2)$, then find f'' in terms of g , g' and g'' .
8. Find the 50th derivative of $y = \cos(2x)$.
9. A particle moves along a straight line with displacement $s(t)$, velocity $v(t)$, and acceleration $a(t)$. Show that

$$a(t) = v(t) \frac{dv}{ds}$$

10. Find $\frac{dy}{dx}$ by implicit differentiation
- (a) $\sqrt{xy} = 1 + x^2y$
 - (b) $y \cos(x) = 1 + \sin(xy)$
 - (c) $y \cos(x) = x^2 + y^2$
11. Find y'' by implicit differentiation
- (a) $9x^2 + y^2 = 9$
 - (b) $x^3 + y^3 = 1$
 - (c) $\sqrt{x} + \sqrt{y} = 1$
12. Find the linearization $L(x)$ of the function at a .
- (a) $f(x) = x^4 + 3x^2$, $a = -1$
 - (b) $f(x) = \sin(x)$, $a = \frac{\pi}{6}$
13. Use a linear approximation (or differentials) to estimate the given number
- (a) $(8.06)^{\frac{2}{3}}$
 - (b) $\frac{1}{4.002}$
14. The radius of a circular disk is given as 24 cm with a maximum error in measurement of 0.2 cm.
- (a) Use differentials to estimate the maximum error in the calculated area of the disk.
 - (b) What is the relative error? What is the percentage error?