

# Math-UA 121: Worksheet Ten

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## Extrema & Their Theorems

1. Find critical points of the following, and determine (if possible) the absolute maximum and minimum of the function in the given interval.

(a)  $f(x) = \ln(x - 2)$ ,  $[0, 5]$

(b)  $g(x) = x^3 + x^2$ ,  $[-2, 4]$

(c)  $h(x) = x^3 - 10x^2 - 7x + 36$   $[-1/2, 1/2]$

(d)  $i(x) = |x - 5|$   $[-6, 4]$

(e)  $j(x) = e^{-x} + \frac{1}{x}$ ,  $[\ln(6), \ln(9)]$

2. Find the values of  $c$  in the given interval that satisfy

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

(a)  $k(x) = 3x^2 + 4x + 2$ ,  $(-4, -1)$

(b)  $l(x) = \sin(2x)$ ,  $(-\frac{\pi}{2}, \frac{\pi}{2})$

(c)  $m(x) = \frac{x^2 + 3x + 2}{x^2 + 7x + 10}$   $(-9, 0)$

(d)  $n(x) = e^{-x}$   $(4, 1)$

(e)  $j(x) = \sqrt{9 - x^2}$ ,  $(10, 15)$

3. Suppose  $f$  is continuous on  $[a, b]$ , differentiable on  $(a, b)$  and  $|a - b| \leq |f(a) - f(b)|$  for all  $a, b \in \mathbb{R}$ . What can you say about  $f(c)$ ?
4. If  $|f'(x)| \leq 4$  for all  $x$ , what is the largest  $f(3) - f(2)$  can be? The smallest?
5. Is it possible, given  $f$  continuous and differentiable everywhere, for  $f'(x) \geq 2$  for all  $x$  it and to have  $f(a) - f(b) < 0$  for all  $a > b$ ? If yes, give an example of such a function.
6. Suppose  $f(x) > 5$  for all  $x < 0$  and  $f'(x) \geq 0$  for all  $x$ . What is the smallest possible value of  $f(0)$ ?