Algebra and Calculus Worksheet: Recitation 3 (9-28-15)

Name: ___________________________

1. **Section 1.8: Question 39**

   Solve the inequality, express the solution in interval notation, and then graph the solution set:

$$x(2x+7) \geq 0$$

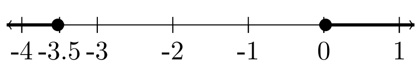
   Solution: First solve the equation

$$x(2x+7) = 0$$
$$\implies x = \left\{0, -\frac{7}{2}\right\}$$

   This is a quadratic equation with two distinct roots, so we expect alternating positive and negative signs. Let's do a sign test:

   - For $x < -\frac{7}{2}$, try x=-4: $(-4)(2(-4)+7) = (-4)(-1) = 4 > 0$. This works.
   - For $-\frac{7}{2} < x < 0$, try x=-1: $(-1)(2(-1)+7) = -1(5) = -5 < 0$. This doesn't work.
   - For $x > 0$, try x=1: $1(2(1)+7) = 1(9) = 9 > 0$. This works.

   So our solution is $\left(-\infty, -\frac{7}{2}\right] \cup [0, \infty)$. Graphically,



2. **Section 1.8: Question 59**

   Solve the nonlinear inequality $\dfrac{x-3}{x+1} \geq 0$, express the solution using interval notation, and then graph the solution set.

   Solution: Before solving the equation to look for critical points, note that this is a problem involving a quotient, so we need to check the denominator to make sure we don't have any singularities. In this case, we do. We set the denominator equal to zero to find the point where this happens:

$$x + 1 = 0$$
$$\implies x = -1$$

   So this will have to be included when we test different regions to see whether or not they satisfy the inequality. Now let's look at the equation:

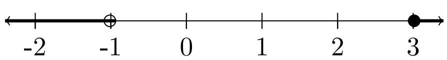
$$\frac{x-3}{x+1} = 0$$
$$\implies x - 3 = 0$$
$$\implies x = 3$$

So now we can divide our number line into three regions: $x < -1$, $-1 < x \leq 3$, and $x \geq 3$. Let's test these regions. Note that because we do not have a strict inequality, the solution to the equation $x = 3$ is part of the solution set. **However**, the singular point $x = -1$ is **not**, as this makes the denominator zero, and dividing by zero is an invalid operation.

Let's test:

- $x < -1$: Let's try x=-2. $\frac{-2-3}{-2+1} = \frac{-5}{-1} = 5 > 0$. This works.
- $-1 < x \leq 3$: Let's try x=0. $\frac{0-3}{0+1} = \frac{-3}{1} = -3 < 0$. This does not work.
- $x \geq 3$: Let's try x=4. $\frac{4-3}{4+1} = \frac{1}{5} > 0$. This works. We could also go by the fact that the equation is satisfied at the point $x = 3$ and recognize that this begins a region where the inequality is satisfied.

The solution set is $(-\infty, -1) \cup [3, \infty)$.



3. Solve the nonlinear inequality $\dfrac{2x+1}{x-5} \leq 3$, express the solution using interval notation, and then graph the solution set.

   Solution: Same procedure as the last problem. First, look for singular points. We have one:

$$\begin{aligned} x - 5 &= 0 \\ \implies x &= 5 \end{aligned}$$

So $x = 5$ is not included in the solution set, and we must use it to divide up our number line. Now, solve the equation:

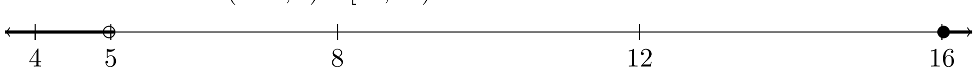
$$\begin{aligned} 2x + 1 &= 3(x - 5) \\ 2x + 1 &= 3x - 15 \\ x &= 16 \end{aligned}$$

So we divide up our number line into: $x < 5$, $5 < x \leq 16$, and $x \geq 16$.

Test:

- $x < 5$: Let's try x=0. $\frac{2(0)+1}{0-5} = -\frac{1}{5} < 3$. This works.
- $5 < x \leq 16$: Let's try x=6. $\frac{2(6)+1}{6-5} = \frac{13}{1} = 13 > 3$. This does not work.
- $x \geq 16$: Let's try x=20. $\frac{2(20)+1}{20-5} = \frac{41}{15} < 3$. This works. We could also go by the fact that the equation is satisfied at the point $x = 16$ and recognize that this begins a region where the inequality is satisfied.

The solution set is $(-\infty, 5) \cup [16, \infty)$.



4. Find the equation of a line with slope 3 whose x-intercept is 3. What is the y-intercept?

   Solution: What is the x-intercept? It is where the line intersects the x-axis. That is the point where y=0. So we know that y=0 when x=3, and that the slope is 3. This is enough to give us the equation:

$$y - 0 = 3(x - 3)$$
$$y = 3x - 9$$

   From the form of the equation, the y-intercept is $b = -9$.

5. Do the lines $y = 3x + 7$ and $y - 7 = 3(x - 4)$ intersect? If so, where? If not, why not?

   Solution: The lines do not intersect because they are parallel. The second line may be written as $y = 3x - 12 + 7 = 3x - 5$. So the two lines are not the same, and they stay parallel without ever intersecting.

6. Do the lines $y = 3x + 7$ and $y - 13 = 3(x - 2)$ intersect? If so, where? If not, why not?

   Solution: The lines are parallel, but they are in fact the **same** line! The second line is: $y = 3x - 6 + 13 = 3x + 7$. So the lines intersect **everywhere**.

7. Sketch the function $f(x) = (x - 1)^2(x + 2)$

   Solution

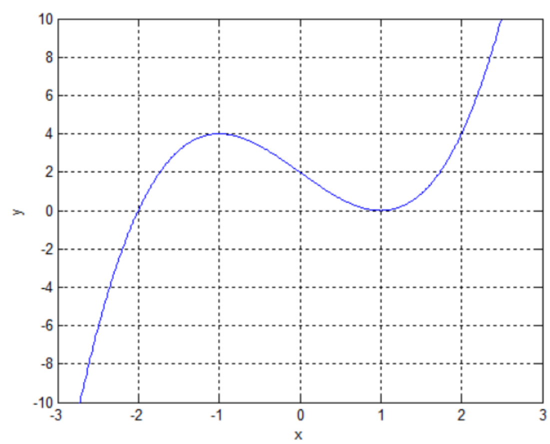


Figure 1: Graph of $(x - 1)^2(x + 2)$

   One can use the sign test to determine where the function is positive and where it is negative.

8. Consider the following two graphs:

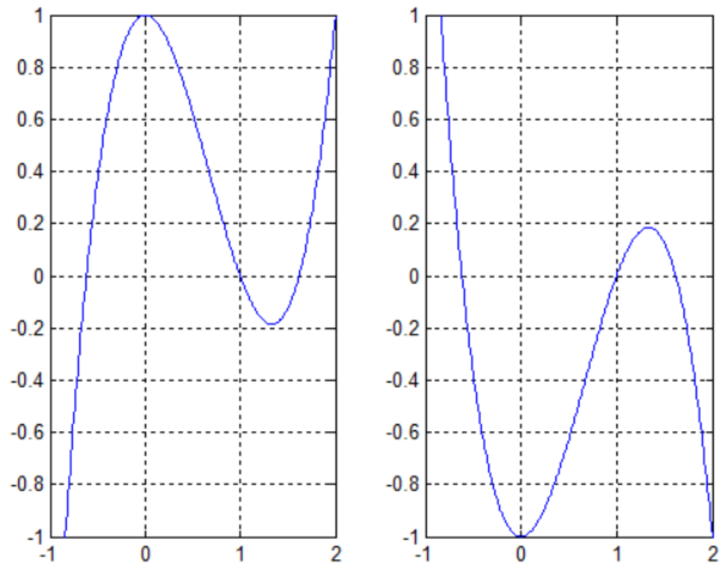


Figure 2: Some graphs of functions.

   (a) Can you guess what kind of polynomials these are? (linear? quadratic? cubic? quartic? higher powers?). Assume the polynomials continue in the same direction beyond the edges of the box.

   (b) Where are the maxima and minima of these functions? Where are the functions increasing and decreasing?

   (c) Can you tell how the two functions are related to each other?

   Solution: These are cubic polynomials. They hit the x-axis three times, we have no repeated roots, and since we know they continue onward without hitting the x-axis again, we can conclude that these are cubics.

   The maxima and minima are located at the crests and troughs (respectively) of the graphs. For the first graph, the maximum and minimum are at $x = 0$ and $x = \frac{4}{3}$, respectively. For the second graph, the locations are switched.

   The two functions are mirror images of each other about the x axis. This is why the minimum and maximum are flipped. They also have the same roots, since they hit the x-axis in the same place.

9. Find the domain of the following functions

   (a) $f(x) = 1 - \sqrt{2x}$

   (b) $f(x) = \sqrt{1 - 2x}$

   (c) $f(x) = \frac{1}{\sqrt{1-2x}}$

   (d) $f(x) = \frac{1}{\sqrt{2x-1}}$

   Solution:

   (a) The term $2x$ is under the square root, which means we need $\boxed{x \geq 0}$ (0 is a perfectly valid part of the domain).

(b) Now $1-2x$ is under the square root, which means we need $1-2x \geq 0 \implies \boxed{x \leq \frac{1}{2}}$.

(c) Now we put $1-2x$ in the denominator. The same restrictions under the square root apply, but now $x = \frac{1}{2}$ is no longer valid. So we need $\boxed{x < \frac{1}{2}}$.

(d) Now we negate the argument under the square root. We need $2x-1 > 0 \implies \boxed{x > \frac{1}{2}}$.

10. Listed for each question below are two quantities. One is the input for a function, and the other is the output for the function.

- Identify which should be the input and which should be the output.
- Consider what the corresponding graph might look like.

(a) the cost to ship a package; the weight of a package

(b) the time of year; the average daily temperature in New York

(c) the probability you decide to go fishing today; the distance you live from a large body of water

Solution: We'll put the **output in bold**, and the *input in italics*:

- **the cost to ship a package**; *the weight of a package*. This would likely be an increasing function (we can't say if it's linear, quadratic, etc.).
- *the time of year*; **the average daily temperature in New York**. This would not be monotonic. We expect a maximum during summer, a minimum during winter, and some intermediate values during spring and fall.
- **the probability you decide to go fishing today**; *the distance you live from a large body of water*. We expect a maximum value at $x = 0$ (since you're most likely to fish if you live right on the water), with the function decreasing as x increases. Note that we can only have nonnegative values of x, since the concept of a negative distance does not make sense.