

Algebra and Calculus Quiz 6

Name/NYU ID:

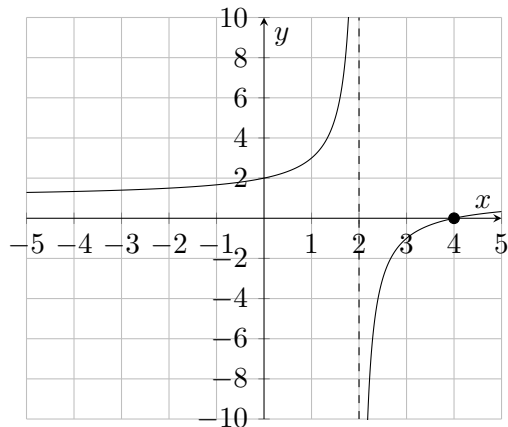
November 2, 2015

Complete all problems.

1. For **multiple choice** problems, circle the letter corresponding to the correct answer.
2. For **true or false** problems, indicate whether you believe the statement is true or false and put a box around your answer (as shown).
3. For **free response** problems, **show all work** and put a box around your final answer.

Good luck!

1. True or false:
 - (a) The graph of the function $f(x) = \frac{1}{x}$ never hits or crosses either axis.
Solution: True. The graph has asymptotes at $x = 0$ and $y = 0$ and never crosses the x or y axis.
 - (b) The line $x = a$ is a vertical asymptote of $y = f(x)$ if $y \rightarrow \pm\infty$ as $x \rightarrow a$ from the left or right.
Solution: True. The situation described above characterizes a vertical asymptote. If drawn on a graph, the function would approach the line $x = a$ from the left and right but never touch it (because $x = a$ is excluded from the domain).
 - (c) A rational function $r(x) = \frac{P(x)}{Q(x)}$ has a slant asymptote if $P(x)$ has the same order as $Q(x)$.
Solution: False. $P(x)$ must be one degree higher than $Q(x)$ for the rational function to have a slant asymptote.
2. Which function does this graph represent?
Hint: Look at the options as transformations of the function $f(x) = \frac{1}{x}$



- (a) $\frac{x-4}{x-2}$
- (b) $\frac{x-2}{x-4}$
- (c) $\frac{2x-4}{x-1}$
- (d) $\frac{2x-1}{x-1}$

Solution: An easy way to do this: we see that the graph has a vertical asymptote at $x = 2$. The only answer choice with a vertical asymptote at $x = 2$ is $\boxed{\text{(a)}}$, which is the answer.

3. The vertical asymptote(s) of $r(x) = \frac{x+100}{x^2-1}$ are:

- (a) $x = 1$
- (b) $x = 1$ and $x = -1$
- (c) $x = -100$
- (d) There is no vertical asymptote.

Solution: To find the vertical symtpotes, set the denominator equal to zero:

$$x^2 - 1 = (x + 1)(x - 1) = 0$$

$$\implies x = \{-1, 1\}$$

And the numerator does not have any factors that cancel with the denominator (i.e. there are no holes in the graph). So the answer is $\boxed{\text{(b)}}$.

4. The horizontal asymptote of $r(x) = \frac{x+100}{x^2-1}$ is:

- (a) There is no horizontal asymptote.

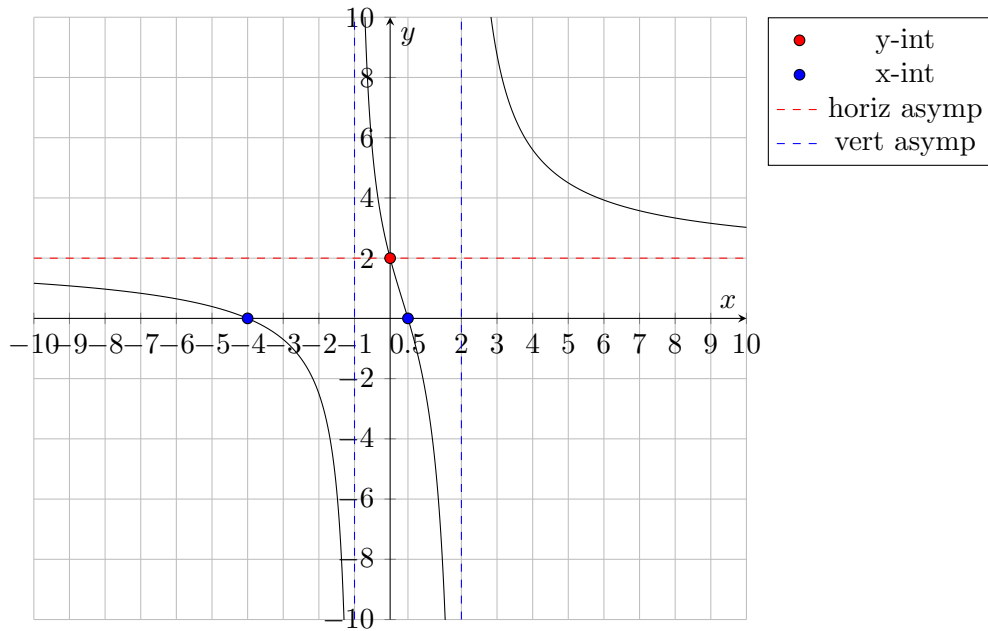
- (b) $y = 0$.
- (c) $y = 1$.
- (d) $y = 100$

Solution: Compare the degree of the numerator and the denominator. We see that the numerator is of degree 1, while the denominator is of degree 2. Thus, the denominator is of higher degree than the numerator, so the function has a horizontal asymptote at $y = 0$. The answer is (b).

5. Graph $f(x) = \frac{2x^2+7x-4}{x^2-x-2}$. Label asymptotes and intercepts.
Hint: Check behavior close to the asymptotes.

Solution:

A graph of the function is below:



There are vertical asymptotes at $x = -1$ and $x = 2$, and a horizontal asymptote at $y = 2$. The best way to draw the graph is to connect the dots; if you determine the asymptotes, the x-intercepts, and the y-intercept, and draw a graph that crosses each axis the appropriate number of times (we have two x-intercepts, so the graph must cross the x-axis exactly twice), then the graph should be correct.