Algebra and Calculus: Quiz 3

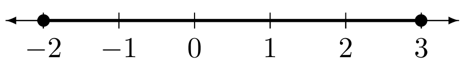
Name/NetID: ____________________________

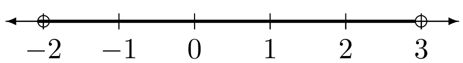
Complete all problems.

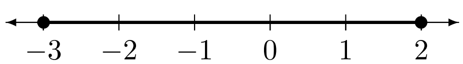
1. For **multiple choice** problems, circle the letter corresponding to the correct answer.
2. For **free response** problems, **show all work** and put a $\boxed{\text{box}}$ around your final answer.

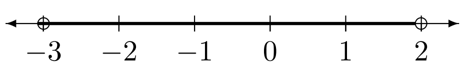
Good luck!

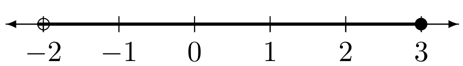
1. Choose the picture that represents the solution set of the inequality: $x^2 - x - 6 < 0$.

(a) 

(b) 

(c) 

(d) 

(e) 

Solution: First, make the inequality into an equation to find the roots:

$$\begin{aligned} x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ x &= \{3, -2\} \end{aligned}$$

Now we can do our usual sign tests. The two roots of the equation divide our number line into three regions: $x < -2$, $-2 < x < 3$, and $x > 3$. Note that because our inequality is a *strict* inequality, we must not include the values of x where the equation is satisfied in our solution.

So let's test our regions:

(a) If $x < -2$, choose $x = -3$: $(-3-3)(-3+2) = (-6)(-1) = 6 > 0$, so this does not work.

(b) If $-2 < x < 3$, choose $x = 0$: $(0-3)(0+2) = (-3)(2) = -6 < 0$, so this works.

(c) If $x > 3$, choose $x = 4$: $(4-3)(4+2) = 6 > 0$, so this does not work.

Thus, our solution region is $-2 < x < 3$. Represented on a number line, this corresponds to choice $\boxed{\text{B}}$.

2. Find the equation of the line perpendicular to $3x + 4y = 5$ at the point $(-1, 2)$.

   (a) $y - 2 = \frac{4}{3}(x+1)$
   (b) $y - 1 = \frac{4}{3}(x+2)$
   (c) $y + 1 = \frac{4}{3}(x-2)$
   (d) $y + 2 = -\frac{3}{4}(x-1)$
   (e) $y - 2 = -\frac{3}{4}(x+1)$

Solution: The first step is to get the equation into the standard slope-intercept form:

$$\begin{aligned} 4y &= 5 - 3x \\ \implies y &= -\frac{3}{4}x + \frac{5}{4} \end{aligned}$$

Now we know that our slope is $-\frac{3}{4}$. If we want to find the equation of a line perpendicular to this, we need to find the negative reciprocal of this slope:

$$\begin{aligned} m &= -\left(-\frac{3}{4}\right)^{-1} \\ &= +\frac{4}{3} \end{aligned}$$

So we know the slope, and we also know a point we would like our line to intersect. So we already have our answer!

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 2 &= \frac{4}{3}\left(x - (-1)\right) \\ y - 2 &= \frac{4}{3}(x+1) \end{aligned}$$

Thus our answer is $\boxed{\text{A}}$.

3. Apple suggests your new laptop should be kept in temperatures between 23 and 95 degrees Fahrenheit. Between which of the following temperatures in degrees **Celsius** should you keep your laptop? *Hint:* $C = \frac{5}{9}(F - 32)$

   (a) 5 and 35

   (b) -5 and 30

   (c) -5 and 35

   (d) -35 and 35

   (e) -35 and 5

   Solution: This simply tests your ability to substitute numbers into a formula. You're given the temperatures in Fahrenheit. Let's substitute the lower limit:

$$\begin{aligned} C_{\text{low}} &= \frac{5}{9}(F_{\text{low}} - 32) \\ &= \frac{5}{9}(23 - 32) \\ &= \frac{5}{9}(-9) \\ &= -5 \end{aligned}$$

   And the upper limit:

$$\begin{aligned} C_{\text{high}} &= \frac{5}{9}(F_{\text{high}} - 32) \\ &= \frac{5}{9}(95 - 32) \\ &= \frac{5}{9}(63) \\ &= 5(7) \\ &= 35 \end{aligned}$$

   Thus, we want to keep our laptop between -5 and 35 degrees Fahrenheit, or choice $\boxed{\text{C}}$.

4. For which values does the inequality $\frac{3}{x^2} - \frac{3}{x^2+1} > 0$ hold?

   (a) All real numbers except -1 and 1.

   (b) All real numbers.

   (c) All real numbers except 0, 1, and -1.

   (d) All real numbers except 0.

(e) No real numbers.

Solution: This is an interesting problem in that it presents you with a statement of fact early on as you work to solve the inequality. First, note the singularity at x=0. We know that x=0 cannot be part of the solution because it is in the denominator in the first term. Look what happens when we try to solve:

$$\begin{aligned} \frac{3}{x^2} - \frac{3}{x^2+1} &> 0 \\ \text{(assuming x is not zero)}\ \frac{3}{x^2}\frac{x^2+1}{x^2+1} - \frac{3}{x^2+1}\frac{x^2}{x^2} &> 0 \\ \frac{3(x^2+1)-3x^2}{x^2(x^2+1)} &> 0 \\ \implies 3x^2 - 3x^2 + 3 &> 0 \\ \implies 3 &> 0 \end{aligned}$$

What can we gather from this statement? Given that x=0 is excluded, this means that we've arrived at a statement of fact. **Any** real value of x besides x=0 will work here!

Another way to see this is to note that the denominator in both terms of the original equation (x^2 and x^2+1) are positive. The denominator in the second term will always be greater than the denominator in the first term (because $x^2+1 > x^2$). Because the two terms share the same numerator, this means that the expression on the left hand side will always be positive, so the inequality is an obvious one in this sense. But again, x=0 will not work.

So the answer is $\boxed{\text{D}}$.

5. Sketch $y = |x+3|$ and $y = 2$. Use the sketch to solve the inequality $2 \geq |x+3|$.

   Solution: For this problem you were being graded on whether or not the solution is immediately deducible from the shaded region of your graph (that is, were you able to solve the problem by graphing the two functions and correctly identifying where the function $y = 2$ exceeded the function $y = |x+3|$. This is one of those cases where you could solve the problem algebraically to confirm whatever is on your graph, but the point of this exercise was to put your sketching skills to the test.

Below is a graph of the two functions:

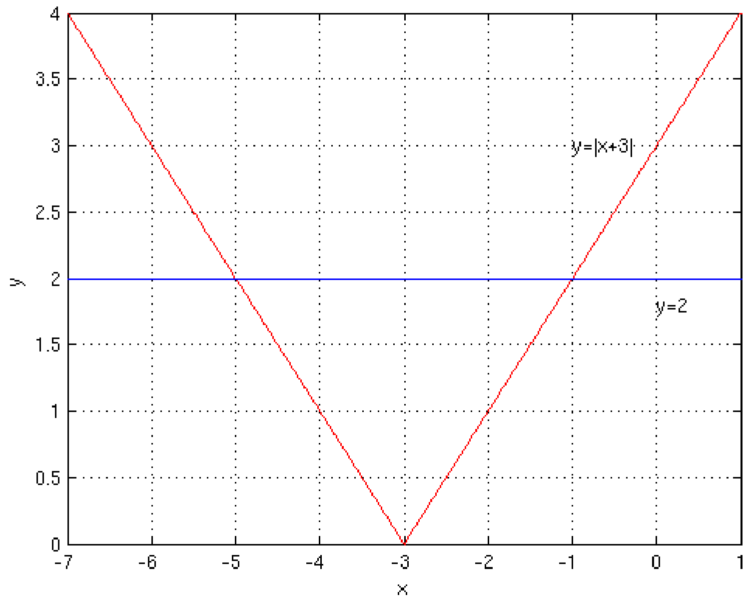


Figure 1: Graph of two functions

We are looking for the values of x in the graph where $|x+3| < 2$. These are the points where the graph of the function $y = |x+3|$ is **below** the graph of the function $y = 2$. This happens when $\boxed{-5 \leq x \leq -1}$