Algebra and Calculus: Quiz 1

Name/NetID: ______________________________

Complete all problems.

1. For **multiple choice** problems, circle the letter corresponding to the correct answer.
2. For **true or false** problems, indicate whether you believe the statement is $\boxed{\text{true}}$ or $\boxed{\text{false}}$ and put a box around your answer (as shown).
3. For **free response**, **show all work** and put a $\boxed{\text{box}}$ around your final answer.

Good luck!

Answers: E, A, C, D, $(1+2x)^{-\frac{3}{2}}$

1. Simplify: $\dfrac{3}{11} - \dfrac{\frac{2}{5}}{\frac{2}{7}-\frac{1}{3}}$.

   (a) $\dfrac{42}{5}$

   (b) $-\dfrac{42}{5}$

   (c) $-\dfrac{477}{55}$

   (d) $\dfrac{21}{11}$

   (e) $\dfrac{477}{55}$

   Solution: The first thing to do is to combine the fractions in the denominator in the term on the right:

$$\frac{2}{7} - \frac{1}{3} = \frac{6-7}{21} = -\frac{1}{21}$$

   Plugging this into the original expression gives us

$$\begin{aligned} &\frac{3}{11} - \frac{\frac{2}{5}}{-\frac{1}{21}} \\ &= \frac{3}{11} + 21\frac{2}{5} \\ &= \frac{3}{11} + \frac{42}{5} \end{aligned}$$

which we may do because dividing by a fraction is equivalent to multiplying by its reciprocal. Now we find the least common denominator between the remaining two fractions and add them:

$$\begin{aligned}\frac{3}{11}+\frac{42}{5} &= \frac{3(5)+42(11)}{55} \\ &= \frac{15+420+42}{55} = \frac{477}{55}\end{aligned}$$

so the answer is $\boxed{\text{E}}$.

2. Factor completely: $(2a+1)^2 - 2(a+1)^2 + 1$

   (a) $2a^2$

   (b) a^2

   (c) $-a^2$

   (d) $-2a^2$

   (e) $2a^2 - 1$

   Solution: Expand all terms:

$$\begin{aligned}(2a+1)^2 - 2(a+1)^2 + 1 &= 4a^2 + 4a + 1 - 2(a^2 + 2a + 1) + 1 \\ &= 4a^2 + 4a + 1 - 2a^2 - 4a - 2 + 1 \\ &= (4-2)a^2 + (4-4)a - 2 + 1 + 1 \\ &= 2a^2\end{aligned}$$

   So the answer is $\boxed{\text{A}}$.

3. Simplify: $\dfrac{1}{x^2-4} - \dfrac{1}{(x+2)^2}$

   (a) $\dfrac{1}{(x+2)^2(x-2)}$

   (b) $\dfrac{2}{(x+2)^2(x-2)}$

   (c) $\dfrac{4}{(x+2)^2(x-2)}$

   (d) $\dfrac{4}{(x+2)^2(x^2-4)}$

   (e) $\dfrac{4}{(x+2)^3}$

   Solution: First, factor the fraction in the denominator of the term on the left:

$$\frac{1}{x^2-4} = \frac{1}{(x+2)(x-2)}$$

   We may now factor out $\dfrac{1}{x+2}$:

$$\frac{1}{x^2-4} - \frac{1}{(x+2)^2} = \frac{1}{(x+2)}\left(\frac{1}{x-2} - \frac{1}{x+2}\right)$$

Now find a common denominator between the fractions in the parentheses:

$$\begin{aligned}\frac{1}{(x+2)}\left(\frac{1}{x-2}-\frac{1}{x+2}\right) &= \frac{1}{(x+2)}\left(\frac{x+2}{(x-2)(x+2)}-\frac{x-2}{(x-2)(x+2)}\right)\\ &= \frac{1}{(x+2)}\left(\frac{x+2-x+2}{(x-2)(x+2)}\right)\\ &= \frac{1}{(x+2)}\left(\frac{4}{(x-2)(x+2)}\right)\\ &= \frac{4}{(x-2)(x+2)^2}\end{aligned}$$

So the answer is $\boxed{\text{C}}$.

4. Eliminate the negative exponents and simplify: $\left(\frac{q^{-1}r^{-1}s^{-3}}{q^{-7}r^{-1}s^{2}}\right)^{-1}$

   (a) q^6s^5

   (b) $\frac{1}{q^6s^5}$

   (c) $\frac{q^6}{s^5}$

   (d) $\frac{s^5}{q^6}$

   (e) $\frac{q^6r}{s^5}$

   Solution: The first step is to flip the terms with negative exponents:

$$\left(\frac{q^{-1}r^{-1}s^{-3}}{q^{-7}r^{-1}s^{2}}\right)^{-1} = \left(\frac{q^{7}r^{1}}{q^{1}r^{1}s^{2}s^{3}}\right)^{-1}$$

   Now we can combine terms:

$$\begin{aligned}\left(\frac{q^{7}r^{1}}{q^{1}r^{1}s^{2}s^{3}}\right)^{-1} &= \left(\frac{q^{7-1}\cancel{r^1}}{\cancel{r^1}s^{2+3}}\right)^{-1}\\ &= \left(\frac{q^6}{s^5}\right)^{-1}\end{aligned}$$

   Lastly, **don't forget to flip the entire fraction!**

$$\left(\frac{q^6}{s^5}\right)^{-1} = \frac{s^5}{q^6}$$

   So the answer is $\boxed{\text{D}}$.

5. Simplify, showing all steps: $\frac{2(1+2x)^{\frac{1}{2}}+(1+2x)^{-\frac{1}{2}}}{8x^2+10x+3}$

Solution: We can start by looking at the fraction in the denominator. This can be factored:

$$8x^2 + 10x + 3 = (2x+1)(4x+3)$$

(one can double check by using FOIL, I solved this by trial and error). Now insert this into the expression, and also factor out the common term $(1+2x)$:

$$\begin{aligned}
&\frac{2(1+2x)^{\frac{1}{2}} + (1+2x)^{-\frac{1}{2}}}{(2x+1)(4x+3)} \\
&= (1+2x)^{-\frac{1}{2}} \frac{2(1+2x)+1}{(2x+1)(4x+3)} \\
&= (1+2x)^{-\frac{1}{2}} \frac{\cancel{4x+3}}{(2x+1)\cancel{(4x+3)}} \\
&= (1+2x)^{-\frac{1}{2}} (2x+1)^{-1} \\
&= (1+2x)^{-\frac{3}{2}}
\end{aligned}$$

So the answer is $\boxed{(1+2x)^{-\frac{3}{2}}}$, or $\dfrac{1}{(1+2x)^{\frac{3}{2}}}$. Both forms are acceptable.