Algebra and Calculus Worksheet: Mega Problem (Midterm 1)

Name: ___________________________

One of the purposes of the initial few weeks of lecture preceding the midterm (in my opinion) is to drive home the point that

1. There are many ways to solve the same problem, BUT
2. Different ways of solving the same problem can lead to different insights and realizations

I'm hoping the following mega problem drives this point home by tying in pretty much everything learned until now.

The Problem

You're a scientist working in a biology laboratory and you're studying two populations of bacteria competing for resources in the same pool, subject to certain conditions (think light input, food sources, etc.). Call one of them population A (P_A), and the other population B (P_B). One of your lab mates ran a previous experiment where she measured the number of species in each population at different times and wrote them into a table. She started with equal amounts of the populations. Here are the values:

Time (hours)	**A**	**B**
0	30	30
2	22	38
4	22	38
6	54	6

1. First, what do you notice about the total number of organisms present in the pool? How does it vary with time?
2. What is the domain for the total length of the experiment? (What is the variable, and on which interval is it defined?). Name one thing that must be true about the range (*Hint: we're dealing with numbers of species*).
3. Your labmate makes a note that the two populations grow according to a **cubic polynomial**, one per population. Find both polynomials. Note that you should be able to confirm your answer to question 1 using your answer to this question.

   Hint: I claim that if you give me four values at four different times, I can give you a unique cubic polynomial that goes through all four points. We are given four values at four different times for each species, and

   $$at^3 + bt^2 + ct + d = \text{Population at time t}$$

   We already know d, as we simply need to plug in t=0 and we get the same number for both populations. Now we have to solve three equations for a, b, and c for each population (so two sets of three interdependent equations).

4. You observe that the two populations start with the same number (30). What is the trend for each population up until t=6 hours? When is population A greater than population B? Vice versa? Use

the table to start, then use the values of your functions P_A and P_B to fill in gaps (e.g. between t=2 and t=4, are the populations actually staying the same for two hours?).

5. From the previous question, you might have noted that at one point, one population is greater than the other, but this switches later on in the table. Thus, we suspect that at some point the two populations become equal again. Find the time where this happens.

   Hint: If the two populations are equal, then $P_A = P_B$. Manipulate the equation to solve for time.

6. Let $F(t) = P_B(t) - P_A(t) = -t(t^2 - 4t - 4)$ be a function representing the difference between the two populations $P_B(t)$ and $P_A(t)$. Where does this equal zero? *Hint: we've done this already.*

7. You decide you would like a graphical representation of the difference between the two populations. Sketch the graph and shade the region where $P_B > P_A$, and write out the solution set. Confirm that this makes sense using the table.

8. Consider the two populations at the time $t = 6$. What is the **average rate** at which each population would have to grow/shrink from time $t = 0$ in order to reach its value at $t = 6$?

9. From your answer to the previous question, explain what is missed by using an average rate of change to explain about how the bacteria grow/decay with time (rely on your answers to previous questions for this). If I only sampled the populations at t=0 and t=6, I would have an entirely different idea of how the populations changed with time. Why?

   Hint: In particular, look at the graph you sketched. Would that region where the inequality is satisfied exist if I only considered the average rates of change?