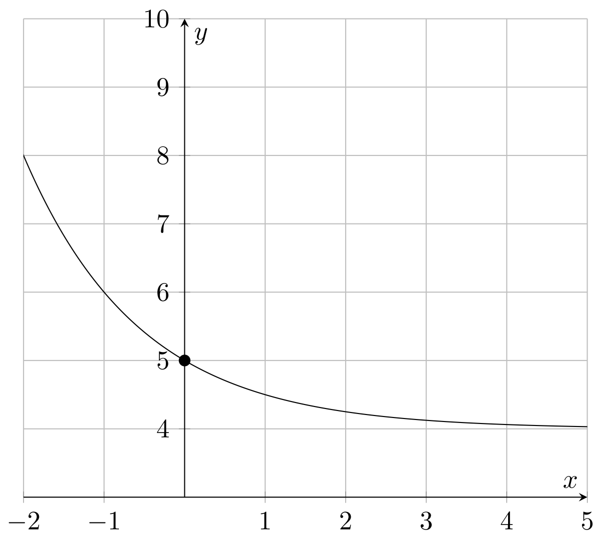
Algebra and Calculus: Homework 9 Solutions

Section 4.1: 28,44

Q28: Graph the function $4 + \left(\frac{1}{2}\right)^x$. State the domain, range, and asymptote.

Solution: From Figure 2 we see that $\left(\frac{1}{2}\right)^x$ decreases as $x \to +\infty$ and increases as $x \to -\infty$. This graph is just the graph of $\left(\frac{1}{2}\right)^x$ shifted 4 units up. It looks like:

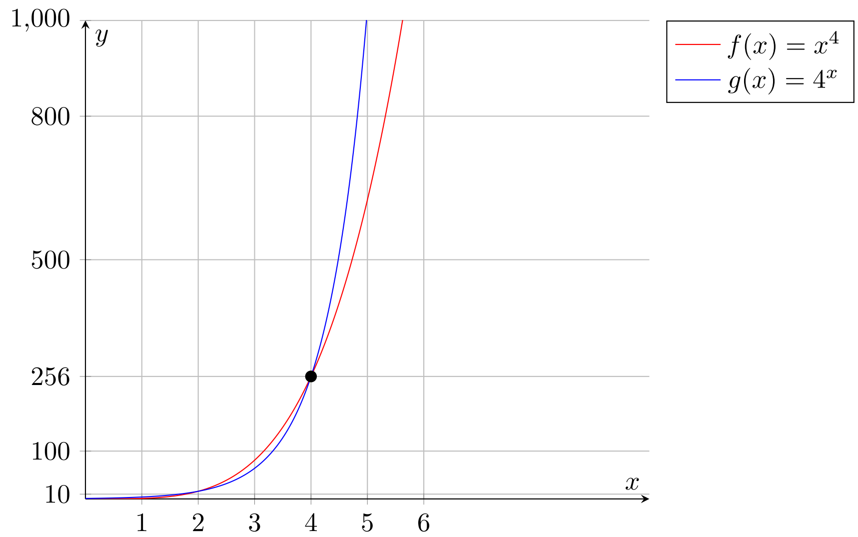


The domain is all $x \in \mathbb{R}$. The range is given by $(4, \infty)$. There is a horizontal asymptote at $y = 4$.

Q44: For the functions $f(x) = x^4$; $g(x) = 4^x$, compare values for $x = 0, 1, 2, 3, 4, 6, 8, 10$, then graph.

Solution: We can make a table:

x	f(x)	g(x)
0	0	1
1	1	4
2	16	16
3	81	64
4	256	256
6	1296	4096
8	4096	65536
10	10000	1048576

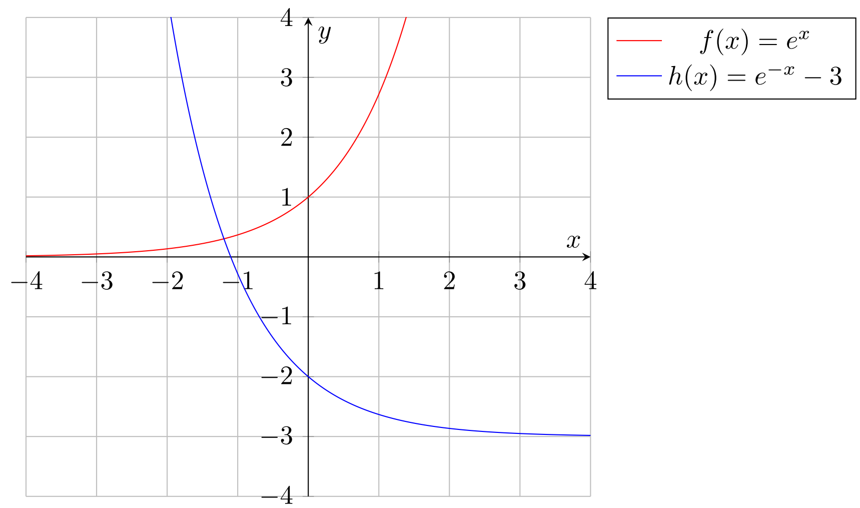


Section 4.2: 8,16

Q8: Graph the function $h(x) = e^{-x} - 3$ by starting from the graph of $y = e^x$. State the domain, range, and asymptote.

Solution: If $f(x) = e^x$, we can write $h(x) = f(-x) - 3$. This means that the graph of $f(x)$ is (1) flipped about the y-axis and (2) shifted down 3 spaces to get $h(x)$.

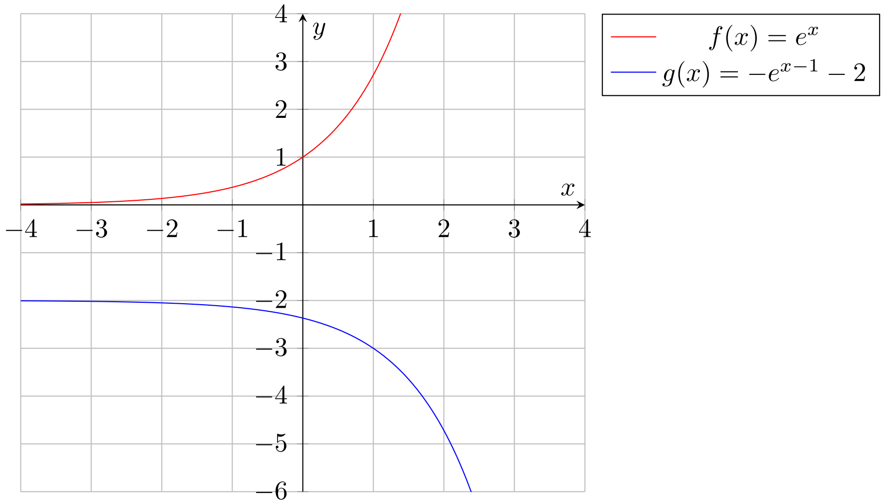
The graph is below:



The domain of $h(x)$ is all real numbers, the range is $(-3, \infty)$ and there is a horizontal asymptote at $y = -3$.

Q16: Same as question 8 except for the function $g(x) = -e^{x-1} - 2$.

We can represent this as $g(x) = -f(x-1) - 2$ with respect to the function $f(x) = e^x$. So we need to take the original graph and (1) shift it one unit to the right, (2) flip it about the x-axis, and (3) move it down two spaces. The graph looks like



The domain is all real numbers, the range is $(-\infty, -2)$ and the horizontal asymptote is at $y = -2$.

Section 4.3: 44,78

Q44: Use the definition of the logarithmic function to find x:

- $\log_x(6) = \frac{1}{2} \implies 6 = x^{\frac{1}{2}} \implies x = 36$
- $\log_x(3) = \frac{1}{3} \implies 3 = x^{\frac{1}{3}} \implies x = 27$

We are simply applying the definition of the log as the inverse of the exponential:

$$c = \log_a(b) \implies b = a^c$$

Q78: Find the domain of $h(x) = \sqrt{x-2} - \log_5(10 - x)$.

Solution: The form of the function implies two constraints on the domain: $x - 2 \geq 0$ (from the square root) and $10 - x > 0$ (from the logarithm). Then combining these inequalities we get that $2 \leq x < 10$.

Section 4.4: 46,56

Q46: Use the laws of logarithms to expand the expression: $\log\left(\frac{x}{\sqrt[3]{1-x}}\right)$.

Solution:

$$\begin{aligned}\log\left(\frac{x}{\sqrt[3]{1-x}}\right) &= \log(x) - \log(\sqrt[3]{1-x}) \\ &= \log(x) - \log\left((1-x)^{\frac{1}{3}}\right) \\ &= \log(x) - \frac{1}{3}\log(1-x)\end{aligned}$$

Q56: Use the laws of logarithms to combine the expression: $2(\log_5(x) + \log_5(y) - 3\log_5(z))$

Solution:

$$\begin{aligned}
2(\log_5(x) + \log_5(y) - 3\log_5(z)) &= 2\log_5(x) + 2\log_5(y) - 6\log_5(z) \\
&= \log_5\left(x^2\right) + \log_5\left(y^2\right) - \log_5\left(z^6\right) \\
&= \log_5\left(x^2y^2\right) - \log_5\left(z^6\right) \\
&= \log_5\left(\frac{x^2y^2}{z^6}\right)
\end{aligned}$$