Calc I: Worksheet (Week 9)

Name: ____________________________

Inverse Trig Functions

1. True or false:
   (a) If $x = \frac{5}{3}\pi$, then $\sin^{-1}(\sin(\frac{5}{3}\pi)) = \frac{5}{3}\pi$
   (b) The range of $\cos^{-1}(x)$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (think about how $\cos^{-1}(x)$ is formed by restricting the domain of $\cos(x)$).
   (c) $\lim_{x\to-\infty} \tan^{-1}(x) = -\frac{\pi}{2}$
2. Find $\frac{d}{dx}\cot^{-1}(x)$ by setting $y = \cot(x)$ and differentiating both sides (this is similar to a homework problem you had).
3. Evaluate the following:
   (a) $\sin^{-1}(\sin(\frac{7}{4}\pi))$
   (b) $\sin^{-1}(\frac{1}{\sqrt{2}})$
   (c) $\arctan(1)$
   (d) $\sin(2\sin^{-1}(\frac{3}{5}))$
4. Prove that $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$ (Hint: Show that $\sin(\sin^{-1}(x) + \cos^{-1}(x)) = 1$, using the formula for $\sin(a + b)$).
5. Find the derivative of:
   (a) $y = \tan^{-1}(x^2)$
   (b) $\sqrt{1 - x^2}\cos^{-1}(x)$
   (c) $x\ln(\tan^{-1}(x))$

L'Hopital's Rule

6. True or False:
   (a) I can use L'Hopital's rule to evaluate $\lim_{x\to3} \frac{e^{x-3}}{x-3}$
   (b) If $f(0) = 0$ and $g(0) = 0$ and f and g are differentiable, then a limit of the form $\lim_{x\to0} \frac{f(x)}{g(x)}$ is equal to the same limit of the *derivative* of $\frac{f(x)}{g(x)}$.
   (c) I can use L'Hopital's rule on $\lim_{x\to\pi} \csc(x) + \cot(x)$.
7. Evaluate the following:

(a) $\lim_{x\to 1} \frac{x^2-1}{x^2-x}$

(b) $\lim_{x\to 0^+} \frac{\ln(x)}{x}$

(c) $\lim_{x\to 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$

(d) $\lim_{x\to 0^+} \sin(x)\ln(x)$

8. Use L'Hopital's rule to show a cool property of the exponential function:

$$e^x = \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n$$

9. Suppose f is a positive function. If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = \infty$, show that

$$\lim_{x\to a} [f(x)]^{g(x)} = 0$$

(this shows that 0^∞ is not an indeterminate form)