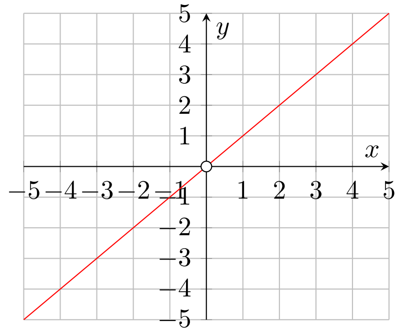
Calc I: Worksheet 1

Name: ______________________________

1. Consider the graph below of the function $f(x)$:



   What is

   (a) $\lim_{x \to 0^+} f(x)$?

   (b) $\lim_{x \to 0^-} f(x)$?

   (c) What can we say about $f(0)$?

   (d) If the only hole in the graph is at $x = 0$, can you think of what the function $f(x)$ may be?

   Hint: Think rational functions.

2. Section 1.3, Q33:

   (Sorry, I had to have at least one example of ϵ, δ theory here)!

   Prove the statement using the ϵ, δ definition of the limit:

$$\lim_{x \to 1} \frac{2 + 4x}{3} = 2$$

3. (a) Does $\lim_{x \to 0} x$ exist? If so, what is the limit?

   (b) Does $\lim_{x \to 0} |x|$ exist? If so, what is the limit?

(c) Does $\lim_{x \to 0} \frac{|x|}{x}$ exist? If so, what is the limit?

4. Compute the following limits:
   (a) $\lim_{x \to 0} \frac{\sin(x)}{x}$
   (b) $\lim_{x \to 0^+} \frac{\sin(x)}{\sqrt{x}}$
   (c) $\lim_{x \to 0} \frac{\sin(x)}{x^2}$

5. Section 1.4, Q19

   Evaluate the limit, if it exists:

$$\lim_{x \to -2} \frac{x+2}{x^3+8}$$

6. Section 1.4, Q22

   Evaluate the limit, if it exists:

$$\lim_{u \to 2} \frac{\sqrt{4u+1}-3}{u-2}$$

7. Section 1.4, Q35

   Prove that

$$\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$$

8. Section 1.4, Q51

   Find the limit.

$$\lim_{t \to 0} \frac{\tan(6t)}{\sin(2t)}$$

9. Section 1.4, Questions 63 and 64

   (a) Show by means of an example that $\lim_{x \to a} [f(x) + g(x)]$ may exist even though neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ exists.

   (b) Show by means of an example that $\lim_{x \to a} [f(x)g(x)]$ may exist even though neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ exists.

10. Section 1.4, Q65

    Is there a number a such that

$$\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

    exists? If so, find the value of a and find the limit.