Algebra and Calculus Worksheet 2

September 21, 2015

1. **Section 1.5 - 59**

   Find all real solutions of the equation by completing the square.

   $$x^2 - 6x - 11 = 0$$

   Solution:

   $$x^2 - 6x - 11 \Rightarrow x^2 - 6x = 11 \Rightarrow x^2 - 6x + 9 = 11 + 9 \Rightarrow (x-3)^2 = 20$$

   Taking the square root of both sides and subtracting 3, we find:

   $$x = 3 \pm \sqrt{20}$$

   This can also be written $x = 3 \pm 2\sqrt{5}$.

2. **Section 1.5 - 71**

   Find all real solutions of the quadratic equation.

   $$3x^2 + 6x - 5 = 0$$

   Solution: Using the quadratic formula, we have

   $$x = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3}$$
   $$\Rightarrow x = \frac{-6 \pm \sqrt{96}}{6}$$
   $$\Rightarrow x = -1 \pm \frac{\sqrt{16 \cdot 6}}{6}$$
   $$\Rightarrow x = -1 \pm \frac{2\sqrt{6}}{3}$$

3. **Section 1.5 - 113**

   Find all real solutions of the quadratic equation.

   $$|3x + 5| = 1$$

   Solution: We have two cases – $3x + 5 = 1$ or $3x + 5 = -1$. Solving each equation for x, we find for the first equation, $x = -\frac{4}{3}$. For the second, we find $x = -2$.

4. **Section 1.7 - 49**

   Ali paints with watercolors on a sheet of paper 20 in. wide by 15 in. high. He then places this sheet on a mat so that a uniformly wide strip of the mat shows all around the picture. The perimeter of the mat is 102 in. How wide is the strip of the mat showing around the picture?

   Solution: This problem is easiest to solve if we look at a drawing – (draw on the board). The length of the mat is $20 + 2x$. The width is $15 + 2x$. The perimeter is:

   $$102 = 2 \cdot (20 + 2x) + 2 \cdot (15 + 2x)$$
   $$\Rightarrow 102 = 40 + 4x + 30 + 4x$$
   $$\Rightarrow 32 = 8x \Rightarrow 4 = x$$

5. **Section 1.7 - 91**

   A 10-ft-long stem of bamboo is broken in such a way that its tip touches the ground 3 ft from the base of the stem. What is the height of the break?

   Solution: First, draw the problem. A triangle is created between the ground, and each broken length of bamboo. The length of the height, call that x, and the length of the hypotenuse (the top of the bamboo that broke off), call that y, sum to 10. In addition, since this is a triangle we have Pythagoras' Theorem, such that $y^2 = x^2+3^2$. Since we have two equations in two variables, we can solve the system (we're asked for the height so solve for x):

   $$x + y = 10; y^2 = x^2 + 9$$
   $$\Rightarrow (10 - x)^2 = x^2 + 9 \Rightarrow 100 - 20x = 9$$
   $$\Rightarrow x = 4.55$$

6. **Section 1.8 - 53**

   Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

$$(x-4)(x+2)^2 < 0$$

Solution: Since the roots of the left hand side of the inequality are 4 and -2, we can remove these from the solution set because the function is equal to 0 at these points. They do, however, tell us the regions we need to test: $(-\infty, -2), (-2, 4),$ and $(4, \infty)$. Pick an easy point to evaluate in each of the regions to find that the function is less than 0 only in $(-\infty, -2) \cup (-2, 4)$. Draw this set on the number line.

7. **Section 1.8 - 69**

   Solve the nonlinear inequality. Express the solution using interval notation, and graph the solution set.

   $$\tfrac{6}{x-1} - \tfrac{6}{x} \geq 1$$

   Solution:

   $$\tfrac{6}{x-1} - \tfrac{6}{x} = \tfrac{6x-6(x-1)}{x(x-1)} = \tfrac{6}{x(x-1)} \geq 1$$

   At this point we realize the function has a singularity such that $x \neq 0, 1$. We'll use this later. Continuing on:

   $$6 \geq x(x-1) \Rightarrow 0 \geq x^2 - x - 6 = (x-3)(x+2)$$

   Our regions to test this inequality are $(-\infty, -2], [-2, 0), (0, 1), (1, 3], [3, \infty)$. The inclusive brackets for the roots of the function are because this time our inequality is not strict. Test points in each region to find that the 0 is less than or equal to the function in $[-2, 0) \cup (1, 3]$. Draw this set on the number line.

8. **Section 1.8 - 83**

   Solve the nonlinear inequality. Express the solution using interval notation, and graph the solution set.

   $$|3x-2| \geq 5$$

   Solution: We have two cases: $3x - 2 \geq 5$ and $3x - 2 \leq -5$. Solving we find $x \geq \frac{7}{3}$ and $x \leq -1$. The solution set is therefore $(-\infty, -1] \cup [\frac{7}{3}, \infty)$. Draw this on the number line.

9. **Section 1.10 - 49**

   Find an equation of the line that satisfies the given conditions.

   Through $(1, 7)$; parallel to the line passing through $(2, 5)$ and $(-2, 1)$.

   Solution: The slope, m is equal to $\frac{rise}{run}$ between points $(2, 5)$ and $(-2, 1)$. So $m = 1$. Then use the point slope formula with the point on the line we are given:

   $$y - 7 = 1 \cdot (x - 1) \Rightarrow y = x + 6.$$

10. **Section 1.10 - 75**

    The equations of two lines are given. Determine whether the lines are parallel, perpendicular, or neither.

    $$-3x + 4y = 4;\ 4x + 3y = 5$$

    Solution: The slope of the first equation is $\frac{3}{4}$ since we can simplify the equation to $y = \frac{3}{4}x + 1$. The slope of the second equation is $-\frac{4}{3}$ since we can simplify the equation to $y = -\frac{4}{3}x + \frac{5}{3}$. One slope is the negative reciprocal of the other so the lines are perpendicular.