Algebra and Calculus: Quiz 2

Name/NetID: ____________________________

Complete all problems.

1. For **multiple choice** problems, circle the letter corresponding to the correct answer.
2. For **true or false** problems, indicate whether you believe the statement is $\boxed{\text{true}}$ or $\boxed{\text{false}}$ and put a box around your answer (as shown).
3. For **free response** problems, **show all work** and put a $\boxed{\text{box}}$ around your final answer.

Good luck!

Answers: False, True, False, C, D, B, "4 problems"

1. For the following three problems, answer **true or false**:

   (a) $\frac{a}{x} + bx = 0$ is a linear equation.

   Solution: False. A linear equation can only have terms that are either (1) a constant or (2) a constant multiplied by x. In this case, we have the term $\frac{a}{x}$, which does not fall under either of these categories. Thus, the equation is not linear.

   (b) If the discriminant of the quadratic equation $ax^2 + bx + c = 0$ is greater than zero, then there are two distinct real solutions.

   Solution: True. The discriminant D comes from the quadratic formula:

   $$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a} \\ D &= b^2 - 4ac \end{aligned}$$

   If D is greater than zero, then because we have a $\pm$ sign in front of a non-zero, real value of $\sqrt{D}$, we will have two unique, real solutions to the equation.

   Recall that if D were zero, we would only have one solution, and if D were negative, then our solutions would be complex.

   (c) There are two distinct real solutions to the equation $(x + 2)^2 - 2x - 3 = 0$.

   Solution: False. This can be factored as $(x + 1)^2 = 0$, which tells us immediately that the only solution is $x = -1$. One can also use the discriminant, as $b^2 - 4ac = 2^2 - 4(1)(1) = 4 - 4 = 0$. Thus there is only one real solution.

   The details:

   $$\begin{aligned} (x + 2)^2 - 2x - 3 &= x^2 + 4x + 4 - 2x - 3 \\ &= x^2 + 2x + 1 = (x + 1)^2 \end{aligned}$$

2. Kim is interested in renting a tiny rectangular studio apartment in SoHo with an area of 72 ft^2. She measures the dimensions and determines that the length exceeds the width by 6 feet. What is the length of the studio?

   (a) 9 ft.

   (b) 15 ft.

   (c) 12 ft.

   (d) 27 ft.

   (e) 28 ft.

   Solution: The length of the studio is 12 ft. One can set this up as a system of equations. We know that the area is equal to the product of the length and the width:

   $$A = lw$$
   $$lw = 72$$

   And we know that the length is 6 feet longer than the width:

   $$l = w + 6$$

   So we have enough information to solve the equation. Let's substitute $w + 6$ for l in $lw = 135$:

   $$(w+6)w = 72$$
   $$w^2 + 6w = 72$$
   $$w^2 + 6w - 72 = 0$$

   One could either guess the factors or use the quadratic formula. Here we factor:

   $$(w+12)(w-6) = 0$$
   $$w = \{-12, 6\}$$

   Clearly the width cannot be a negative number, so the width must be 6, which means that the length must be 6+6 = 12 feet, for answer choice $\boxed{\text{C}}$.

3. John has taken three exams in his history class so far. His exam grades are: 76, 89, and 93. John has one more exam left to take. Assuming his final grade is based entirely on the average of his exam grades, what grade must he earn on his last exam to receive an exact final grade of 88 in the class?

   (a) 88

   (b) 85

   (c) 72

   (d) 94

   (e) 100

   Solution: The formula to calculate an average is given by:

   $$s_{avg} = \frac{s_1 + s_2 + ... + s_n}{n}$$

where n is the number of exams taken, s_{avg} is the average score, and s_1 is test number 1; s_2 is test number 2, etc. In our case, n=4, since there are four tests in total that make up the final grade. We know:

$$\begin{aligned} s_1 &= 76 \\ s_2 &= 89 \\ s_3 &= 93 \\ s_{\text{avg}} &= 88 \end{aligned}$$

So we just need to solve for s_4:

$$\begin{aligned} s_{avg} &= \frac{s_1 + s_2 + s_3 + s_4}{4} \\ 88 &= \frac{76 + 89 + 93 + s_4}{4} \\ 352 &= 76 + 89 + 93 + s_4 \\ s_4 &= 352 - (76 + 89 + 93) \\ s_4 &= 352 - 258 \\ s_4 &= 94 \end{aligned}$$

So he needs a 94, making the correct answer $\boxed{\text{D}}$.

4. Which of the following describes the solution set to the equation: $x^4 - 3x^2 + 2 = 0$?
   (a) $x = -1, x = 1, x = \sqrt{2}$
   (b) $x = -1, x = 1, x = \sqrt{2}, x = -\sqrt{2}$
   (c) All $x > 0$
   (d) $x = 1, x = -3, x = 2, x = \sqrt{2}$
   (e) $x = 1, x = 3, x = -2, x = \sqrt{2}$

   Solution: One way to approach this problem is to simply plug in answers from the multiple choice suggestions, but this is dangerous because all of the choices in (a) will work, but this is not the complete solution set. It is easy enough to factor. Let $b = x^2$:

$$\begin{aligned} x^4 - 3x^2 + 2 &= b^2 - 3b + 2 = 0 \\ (b-2)(b-1) &= 0 \\ b &= \{1, 2\} \end{aligned}$$

   Thus we know that $x^2 = 1$ and $x^2 = 2$. We can now take the square root of both sides and **make sure we include both the positive and negative solutions**. Thus,

$$x = \{-1, 1, -\sqrt{2}, \sqrt{2}\}$$

   making the correct answer $\boxed{\text{B}}$.

5. Irena and Qiu are studying together for an Algebra and Calculus exam. They decide to divide up a set of 6 problems. It would take Irena 9 minutes to complete all the problems on her own, while it would take Qiu 18 minutes to complete all the problems on his own. When they collaborated, it took them just 6 minutes to complete all 6 problems.

   Assuming that nobody took any breaks and they were working simultaneously, how many problems did Irena complete?

   Hint: They're working **simultaneously**, which means that their combined *rate* of problem-solving should be greater than the rate at which they would complete problems if they took turns (i.e. if Irena worked on a problem while Qiu waited for her to finish, which would be inefficient).

   Solution: This is one of those "time needed to do a job" problems. Let n_i equal the number of problems Irena completed and n_q equal the number of problems Qiu completed. We know that

$$n_i + n_q = 6$$

   since there are 6 problems total. We can calculate the rate at which each student works.

   - If it takes Irena 9 minutes to do 6 problems, this means she works at a rate of $\frac{2}{3}$ problems per minute
   - If it takes Qiu 18 minutes to do 6 problems, this means he works at a rate of $\frac{1}{3}$ problems per minute

   This means that after 6 minutes, Irena will have completed 4 problems and Qiu will have completed 2 problems, so it takes them six minutes total, and Irena completed $\boxed{\text{4 problems}}$.

   We can also look at their combined rates:

$$r = \frac{2}{3} + \frac{1}{3} = 1 \text{ problem per minute}$$

   So we see that together they complete a problem per minute, which tells us that it will take 6 minutes for them to complete all 6 problems. Because we know that Irena's work rate is $\frac{2}{3}$ problems per minute,

$$\begin{aligned} n_i &= (6 \text{ minutes})\left(\frac{2}{3}\frac{\text{problems}}{\text{minute}}\right) = 6 \times \frac{2}{3} \text{ problems} \\ &= 4 \text{ problems} \end{aligned}$$