Algebra and Calculus: Mock Quiz 1

Name/NetID: ___________________________

Complete all problems.

1. For **multiple choice** problems, circle the letter corresponding to the correct answer.
2. For **free response** problems, **show all work** and put a $\boxed{\text{box}}$ around your final answer.

Good luck!

1. I have a function such that $(2f \circ f \circ f \circ f \circ f)(x) = 64x$. Which of the following could be the function?

   (a) $f(x) = x^2$
   (b) $f(x) = \frac{x^2}{2}$
   (c) $f(x) = 3x$
   (d) $f(x) = 32x$
   (e) $f(x) = 2x$

   Solution: The easiest way to do this is to simply test the multiple choice answers.

   The answer is $\boxed{\text{E}}$:

$$\begin{aligned}(2f \circ f \circ f \circ f \circ f)(x) &= (2f \circ f \circ f \circ f)(2x) \\ &= (2f \circ f \circ f)(2(2x)) = (2f \circ f \circ f)(4x) \\ &= (2f \circ f)(2(4x)) = (2f \circ f)(8x) \\ &= 2f(2(8x)) = 2f(16x) \\ &= 2(2)(16x) = 64x\end{aligned}$$

2. I have two functions, f(x) and g(x), with the following properties:

$$\begin{aligned}f(x) + g(x) &= 2x^2 - 3 \\ f(x) - g(x) &= 6x + 5\end{aligned}$$

   What values of x solve the equation $g(x) = 0$?

(a) $x = \{-1, 4\}$

(b) $x = \{-3, 2, 5, 6\}$

(c) $x =$ all real numbers

(d) $x = \{1, -3\}$

(e) $x = \{2, 5\}$

Hint: Use the two equations given above to find g(x), then set it equal to 0 and solve in the usual way.

Solution: We are given two equations and want to first solve for g(x). All we need to do is subtract the second equation from the first:

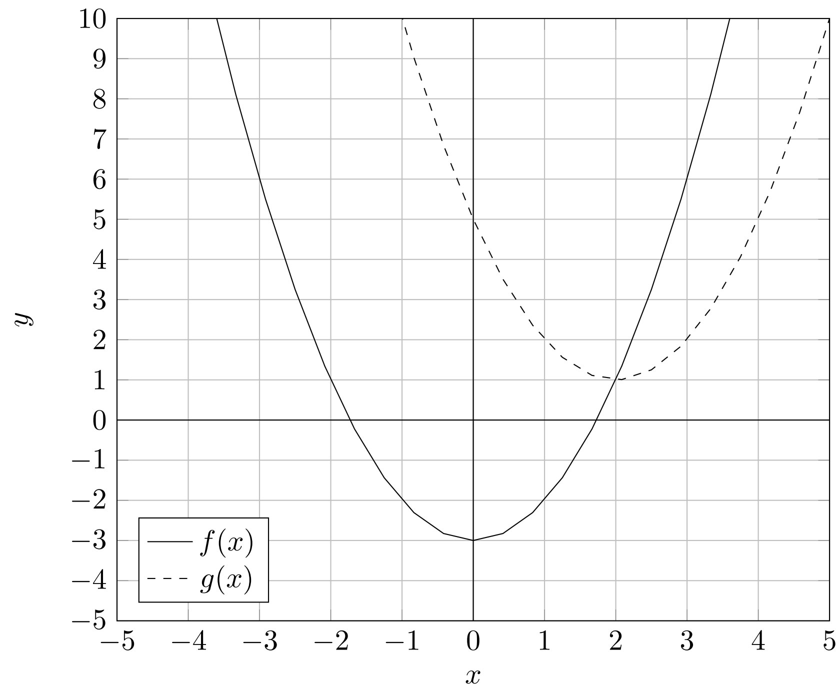
$$\begin{aligned} f(x) + g(x) &= 2x^2 - 3 \\ f(x) - g(x) &= 6x + 5 \\ f(x) + g(x) - (f(x) - g(x)) &= 2x^2 - 3 - (6x + 5) \\ 2g(x) &= 2x^2 - 6x - 8 \\ \implies g(x) &= x^2 - 3x - 4 \end{aligned}$$

Now we just set this equal to 0 and find the roots:

$$\begin{aligned} x^2 - 3x - 4 &= 0 \\ (x - 4)(x + 1) &= 0 \\ x &= \{-1, 4\} \end{aligned}$$

So the answer is $\boxed{\text{A}}$.

3. Consider the graphs of the two functions $f(x)$ and $g(x)$ below:



Assume that $g(x)$ may be represented as $f(x)$ plus one or more shifting operations (i.e. there is no scaling involved). How would I represent $g(x)$ in terms of $f(x)$?

(a) $g(x) = f(x + 4) - 2$
(b) $g(x) = f(x - 2) + 4$
(c) $g(x) = f(x + 2) - 4$
(d) $g(x) = f(x - 2) - 4$
(e) $g(x) = f(x + 4) + 2$

Solution: It turns out the two graphs are:

$$f(x) = x^2 - 3$$
$$g(x) = (x - 2)^2 - 3 + 4 = f(x - 2) + 4$$

so the answer is $\boxed{\text{B}}$. How would one get this from the graph? Note that $f(x)$ is the blue graph and $g(x)$ is red. We know that only shifting operations are involved, so there must be one horizontal shift and one vertical shift. We see from the graph that we must move $f(x)$ 2 spaces to the right and 4 spaces up in order to get to $g(x)$. So:

- 2 spaces to the right $\rightarrow f(x - 2)$

- 4 spaces up $\rightarrow f(x)+4$
- 2 spaces to the right **and** 4 spaces up $\rightarrow f(x-2)+4$

4. Say you're driving a car on a four hour trip with your friend and the distance you've traveled thus far is represented by the function:

$$s(t) = -5t^2 + 40t$$

where the distance is in miles and the time is in hours. Thus, at time t=0, you haven't traveled anywhere, so your distance is $s(0) = 0$. Your velocity is slowing down with time, as you start hitting traffic on your way (we've modeled the distance function such that this is true). After how many hours will your **average** velocity, taken from time t=0 up to the current time, hit 25 miles per hour?

(a) 1 hour

(b) 1.5 hours

(c) 2 hours

(d) 2.5 hours

(e) 3 hours

Hint: What's another name for the the average rate of change of the distance you've traveled?

Solution: The key is to use the average rate of change formula. At a given time t,

$$v_{\text{avg}}(t) = \frac{s(t)-s(0)}{t} = \frac{s(t)}{t} = 40 - 5t$$

And we want to know after how many hours this average velocity will hit 25 miles per hour. Thus, the equation is simple:

$$40 - 5t = 25$$
$$t = \frac{1}{5}(40-25) = \frac{15}{5} = 3 \text{ hrs}$$

So the answer is $\boxed{\text{E}}$.

5. Xinyang and Kevin are playing cards. Xinyang initially puts in 50 dollars and Kevin puts in 20. Xinyang is not doing her best tonight, and has been losing 5 dollars per game to Kevin. Continuing this trend, Kevin and Xinyang share the same amount of money after the conclusion of game g. However, from this point on, Xinyang begins to win money back at a steady rate of 3 dollars per game. **They play 10 games in total**.

(a) Write the piecewise function corresponding to the amount of money Kevin has as a function of the number of games he has played. Make sure you have the right domain based on the total number of games he plays.

(b) How many games will Xinyang and Kevin have played in total before Kevin ends up with 23 dollars?

A graph is provided below if you choose to use it.

Solution: Algebraically, we can solve in two parts. First, we solve for g, the game after which Kevin and Xinyang share the same amount of money:

$$\begin{aligned} 50 - 5g &= 20 + 5g \\ 10g &= 30 \\ g &= 3 \end{aligned}$$

The first equation above says that Xinyang starts with 50 and loses 5 dollars per game, while Kevin gains 5 dollars per game and began with 20. After 3 games, they are even. How much money do they have?

$$50 - 5(3) = 20 + 5(3) = 35$$

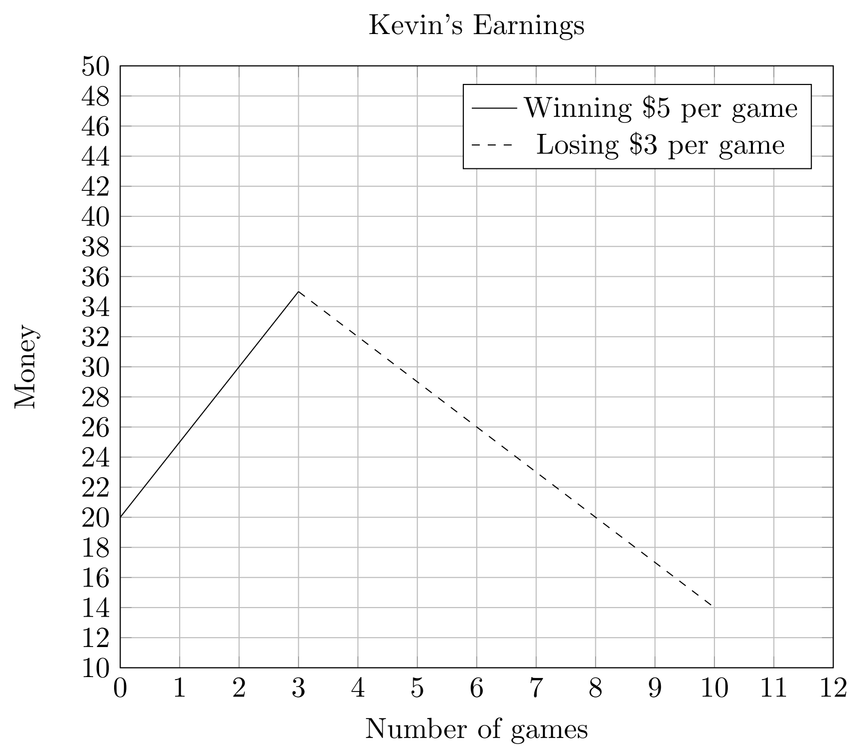
So they have 35 dollars after this game. Now we can solve for g_2, the game after which Kevin has 23 dollars. Recall Xinyang wins 3 dollars per game, which means Kevin loses that amount per game:

$$\begin{aligned} 35 - 3g_2 &= 23 \\ 3g_2 &= 12 \\ g_2 &= 4 \end{aligned}$$

Then 4 games after Xinyang and Kevin tie, Kevin is left with 23 dollars. Thus, **in total**, $g + g_2 = 3 + 4 = 7$.

So the answer to part (b) is $\boxed{\text{7 games}}$.

Graphically, Kevin's plot should look like this:



We see that after 3 games, the two players have the same amount of money. After 7 games total, Kevin has 23 dollars.

This graph corresponds to the piecewise function (i.e. the answer for part (a)):

$$f(x) = \begin{cases} 20 + 5x & x \leq 3 \\ 44 - 3x & 3 \leq x \leq 10 \end{cases}$$

So that is the answer to (a).